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# Schwarzschild's Metric Allows Possibility of Discrete Space-time

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**Schwarzschild's Metric (*1*) under specific conditions provides a first order discrete length when transforming coordinates. In this paper, the discrete length's consequences are explored. The results show an a priori deduction of quantized space-time. Base units derived from the quantization in comparison to Planck's units have many similarities. With elementary charge applied, exact formulations for the observed Schwarzschild's discrete units are obtained. The units equal Planck's mass, length, time, momentum, force, energy and Planck's constant (*2*).**

## Introduction

The Schwarzchild's Metric, the first exact solution to Einstein's field equations of General Relativity is (*1*)

$$c^2 d\tau^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + sin^2\theta d\phi^2\right) \tag{1}$$

where c is the speed of light, $\tau$ is proper time, G is Newton's universal gravitational constant, r is the radial distance from the mass generating the field and t is the time coordinate located an infinite distance from the mass. Consider only a radial change in position of an observer. The transformation of coordinate r, with coordinates $t, \theta, \phi$ unchanged, given by the metric is

$$c^2 d\tau^2 = -\left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 \tag{2}$$

Set

$$x = \frac{M}{r} \tag{3}$$

and perform a Taylor series expansion about zero on x and replace x with $M/r$:

$$1 + \frac{GM}{rc^2} + \frac{3G^3M^2}{2r^2c^4} + O(x^3) \tag{4}$$

A first order approximation of length L by an observer an infinite distance from the mass, transformed to an observer radial distance r from the mass is:

$$L' = \left(1 + \frac{GM}{rc^2}\right) L \tag{5}$$

where L' is the observed length at radial distance r. When

$$r = L \tag{6}$$

equation 5 reduces to

$$L' = L + \frac{GM}{c^2} \tag{7}$$

From equation 6 condition L' is observed with an additional length that is independent of radial distance. Equation 6 allows equation 7 to be written as

$$L' = 2L = \frac{2GM}{c^2} \tag{8}$$

In other words, a distance of $GM/c^2$ an infinite distance from the mass is observed as $2GM/c^2$ locally at the distance $GM/c^2$. Reversing the transformation, where

$$L' = r = \frac{GM}{c^2} \tag{9}$$

The transformation takes the form

$$L = \frac{L'}{2} = \frac{GM}{2c^2} \tag{10}$$

Equation 10 $L'$ is the length $GM/c^2$ as observed at the same radial distance from the mass transformed to an observer infinitely far away. The length is observed as half the length. Recall, this is a first term approximation. Note the cancellation of $r$ in equation 5 of the first term approximation only cancels in the first term. If the approximation is obtained to higher terms, the additional approximations are radial dependent. However, in the higher term approximations the first term approximation will always be independent of radial distance with the condition of equation 6 applied.

This paper's focus is on the value $GM/c^2$. Base units derived from this value and the invariant speed of light are examined. The consequences of the examination are many of Planck's units and Planck's reduced constant $\hbar$.

# Deriving Base Units

Consider a point mass M of unknown magnitude. Derive a unit length from equation 7.

$$\hat{l} = \frac{GM}{c^2} \tag{11}$$

The unit of time is constructed from the duration of a ray of light to traverse the path of unit length.

$$\hat{t} = \frac{GM}{c^3} \tag{12}$$

By examination of equations 11 and 12, the unit velocity is $c$. Unit momentum is

$$\hat{p} = Mc \tag{13}$$

Unit force is constructed from Newtonian gravitational force.

$$\hat{F}_g = \frac{GMM}{\hat{l}^2} \tag{14}$$

Unit Energy is constructed from unit force applied over unit distance.

$$\hat{E}_g = \hat{F}_g \cdot \hat{l} = Mc^2 \tag{15}$$

A unit of energy delivered for a unit of time is

$$\hat{E}_g \hat{t} = \frac{GMM}{c} \tag{16}$$

Equation 16 is the minimum energy delivered over the minimum time. Thus, $\hat{E}_g \hat{t}$ is a quanta of energy exchange in the derived units.

# Elementary Charge Applied

Consider two elementary charges $\hat{l}$ distance apart. The Coulomb force between the two elementary charges is

$$\hat{F}_e = \frac{e^2}{4\pi\epsilon_0 \hat{l}^2} \tag{17}$$

where $\epsilon_0$ is the vacuum permittivity. The force applied over the distance of $\hat{l}$ is the Coulomb unit energy.

$$\hat{E}_e = \hat{F}_e \cdot \hat{l} = \frac{e^2}{4\pi\epsilon_0 \hat{l}} \tag{18}$$

The energy relation between equation 15 and 18 is unknown. Consider an unknown constant of proportionality K applied to the ratio of energies to construct

$$1 = k\frac{\hat{E}_e}{\hat{E}_g} \tag{19}$$

rearranging

$$\hat{E}_g = k\hat{E}_e \tag{20}$$

Expanding equation 20 with values for each unit:

$$Mc^2 = k\left(\frac{e^2}{4\pi\epsilon_0\left(\frac{GM}{c^2}\right)}\right) = \frac{ke^2c^2}{4\pi\epsilon_0 GM} \tag{21}$$

One unit of energy $\hat{E}_g$ for one unit of time $\hat{t}$ relates to one unit of Coulomb energy $\hat{E}_e$ for one unit of time $\hat{t}$ via

$$\hat{E}_g\hat{t} = k\hat{E}_e\hat{t} \tag{22}$$

Rearranging equation 22 and substituting values from equations 21 and 12

$$\left(\frac{1}{k}\right)\left(\hat{E}_g\hat{t}\right) = \left(\frac{e^2c^2}{4\pi\epsilon_0 GM}\right)\left(\frac{GM}{c^3}\right) = \frac{e^2}{4\pi\epsilon_0 c} \tag{23}$$

The ratio of forces $\hat{F}_g$ and $\hat{F}_e$ is

$$\hat{F}_g = k\hat{F}_e \tag{24}$$

Using equation 19 to solve for $M$,

$$M = \sqrt{\frac{ke^2}{4\pi\epsilon_0 G}} \tag{25}$$

rearranging

$$\frac{1}{\sqrt{k}}M = \sqrt{\frac{e^2}{4\pi\epsilon_0 G}} \tag{26}$$

Equation 23 and equation 26 are of particular interest, the RHS consist of only known constants. This relationship (equations 23 and 26) assist in deriving the values for the LHS of equation 23.

# Uncertainty in Measurement

Recalling equation 10, an observer encounters a limit in obtaining a measurement. Via equation 7, the minimum additional length is $GM/c^2$. This implies all measurements have an uncertainty of $GM/c^2$. An observer infinitely far away observing the additional length $GM/c^2$ at distance $r = GM/c^2$ from the point mass has an uncertainty of $GM/2c^2$ as shown in equation 10. The units derived from $GM/c^2$ infer an uncertainty in the measurement of energy. Any energy measured over a time will have the following level of uncertainty:

$$\textit{uncertainty in energy measurement} = \frac{\hat{E}_g \hat{t}}{2} = \frac{\left(\frac{GMM}{c}\right)}{2} \tag{27}$$

Another level of uncertainty arises when an observer measures change in distance (unit length $\hat{l}$) and momentum $\hat{p}$.

$$\textit{uncertainty in momentum and position measurement} = \frac{\hat{l}\hat{p}}{2} = \frac{\left(\frac{GMM}{c}\right)}{2} \tag{28}$$

The last area covered in uncertainty of measurement is angular momentum of a single orbiting particle about the point mass. The angular momentum is

$$L = rmv = \hat{l}mc \tag{29}$$

Via the derived units, angular momentum has the same uncertainty as energy.

$$\textit{uncertainty in angular momentum measurement} = \frac{L}{2} = \frac{\hat{l}mc}{2} = \frac{\left(\frac{GMM}{c}\right)}{2} \tag{30}$$

# Hydrogen Electron Orbital Classical Calculation

Equation 16 is considered the quanta of energy exchange via the derived units. Applying this to a classical derivation, similar to Bohr's radius derivation (*3*) , the orbital of an electron about a proton (the Hydrogen atom) is derived. The angular momentum must be in steps of the quanta of angular momentum. Therefore the angular momentum of the electron (classically derived) is:

$$L_{electron} = n(\hat{l}mc) \tag{31}$$

As derived earlier, the constant of proportionality k must be applied when using Coulomb force. The angular momentums (gravitational derived unit force and Coulomb derived unit force) applying equations 20 and 31 when n = 1, is

$$kr_e m_e c = \hat{l}Mc \tag{32}$$

where $m_e$ is the electron mass, $r_e$ is the orbital radius, and the RHS side is derived units. Solving for $r_e$,

$$r_e = \frac{\hat{l}Mc}{km_e c} \tag{33}$$

Equation 33 is a classically derived orbital radius of an electron about a proton. Unknown are the values for $k$ and $\hat{l}Mc$.

# Applying Empirical Data and Results

Constraints have been placed on the derived unit values. Specifically equation 23 where the constants $k$ and $\hat{E}_g\hat{t}$ have an invariant value. Two known constants whose product is the RHS of equation 23 are the fine structure constant and Planck's reduced constant. Consider

$$k = \alpha \tag{34}$$

$$\hat{E}_g\hat{t} = \hbar \tag{35}$$

where $\alpha$ is the fine structure constant. Equation 23 takes the form

$$k\hat{E}_g\hat{t} = \alpha\hbar = \frac{e^2}{4\pi\epsilon_0 c} \tag{36}$$

Substituting these values into the derived values has several results. The first results to consider are

$$M = M_p \tag{37}$$

$$\hat{l} = l_p \tag{38}$$

$$\hat{t} = t_p \tag{39}$$

$$\hat{p} = p_p \tag{40}$$

$$\hat{F}_g = F_p \tag{41}$$

$$\hat{E} = E_p \tag{42}$$

where the subscript p represents Planck's units. The uncertainty in measurement from equations 27, 28 and 30 is:

$$\textit{Uncertainty in measurement} = \frac{\hbar}{2} \tag{43}$$

The last result discussed in this paper is the classical derivation of the electron orbital of the Hydrogen atom. The results from equation 33 is an exact value of Bohr's radius derivation.

# Summary

Schwarzschild's metric provides the possibility of discrete space-time. From the discrete, constant length of $GM/c^2$ a set of base units were derived. These units, after a priori derivation and constraints, provide supporting evidence of deriving Planck's reduced constant. This constant, once derived, provides many paths of other derived (and well known) mechanics. A derived path from a General Relativity exact solution to discrete space-time has been shown. Further investigation, rigor and modeling is needed.

# References and Notes

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